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A stochastic model of spin glass dynamics

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Abstract. A dynamical theory of spin glasses based on the (assumed) morphology of the ground state and on some scaling hypotheses is presented. It yields a closed formula for the AC susceptibility which can reproduce the experimentally observed behaviour of χ in both frequency and temperature. The time-dependent susceptibility is also analysed: it is found that equilibrium is reached in an extremely slow manner, and might never be observed in real experiments.

1. Introduction

The frequency and temperature dependence of the AC susceptibility of real spin glasses is often explained in terms of super-paramagnetic clusters (Lundgren *et al* 1981, van Duyneveldt and Mulder 1982) which, however, are neither accounted for by microscopic theory nor have they been found in numerical experiments. On the other hand, all the theoretical effort spent on mathematical, microscopic models has given much insight into the physics of spin glasses, but yet no prediction is in substantial agreement with the experiments.

The purpose of this paper is to bridge the gap between the theories which start with a Hamiltonian and the experiments; we construct a theory which is 'quasi-phenomenological' in that it is based on the morphology of the ground state of spin glass models and some scaling hypotheses which are so far unproved and yet can (at least in principle) be checked by Monte Carlo calculations or other numerical methods. The theory predicts a frequency and temperature dependence of the AC susceptibility which, for 'small' frequencies is in very good agreement with the experimental results, and can hopefully be further developed into a true microscopic theory of the spin glass phase. We also calculate the time-dependent susceptibility and find that equilibrium probably cannot be reached within the lifetime of the observer.

2. Ground-state morphology and clusters

Exact results on the morphology of the ground state of some simple spin glass models are now available; we conjecture that the same kind of picture also applies to 'real' three-dimensional systems, and elaborate on the dynamical consequences.

The ground-state manifold of a $\pm J$ model on a square lattice was analysed in great detail by Bieche *et al* (1980) and Barahona *et al* (1981) for $T=0$ and different concentrations of antiferromagnetic bonds. When the last quantity is sufficiently large, the spins form connected patches which maintain the same orientation in all the ground states, a feature which is apparent in the nice graphs of Barahona *et al.* The same ground-state structure has been proposed previously (Binder 1976, Bray and Moore 1977, Vannenimus *et al* 1979) in different contexts and on the basis of Monte Carlo simulations, analytical results and hand calculations. More recently, Hertz and Sibani (1984) and Sibani and Hertz (1985) constructed two models with random interactions and an exactly calculable ground-state manifold; both have the aforementioned morphology. The connected patches of spins which have the same relative orientation in all the ground states have been called 'packets of solidary spins'. We shall use the over-worked but shorter term 'cluster' for the same object. A cluster can be flipped as a whole at no energy cost, and corresponds to a local symmetry of the model. We shall now briefly discuss some properties of these clusters, especially the distribution of their sizes, which is closely related to one important concept in the theory of random systems, the Parisi overlap density (Parisi 1983).

Since the set of clusters is clearly closed under union, we have a nested or hierarchical structure of clusters. The smallest clusters, which we call irreducible, label by their orientations the pure states of the system at $T=0$, and their sizes determine the spatial correlations. Indeed, $|\langle\sigma_i\sigma_j\rangle|=1$ if the ith and jth spin belong to the same cluster, and zero otherwise. The Parisi overlap function $P(q)$ is by definition the density of

$$\lim_{N\to\infty}\left(\frac{1}{N}\sum_{i=1}^{N} m_i^\alpha m_i^\beta\right),$$

where α and β are two randomly chosen pure states, m_i^α is the magnetisation at site i in the pure state α, and the sum goes over all the N sites in the lattice. Letting X_i denote the size of the ith irreducible cluster it is easy to show (Sibani and Hertz 1984) that $P(q)$ is the density of the stochastic variable

$$Q=\lim_{n\to\infty}\frac{\lambda_1X_1+\ldots\lambda_nX_n}{X_1+\ldots X_n}, \tag{2.1}$$

where each λ_i is a stochastic variable with equiprobable outcomes ±1. The X_i's depend on the couplings and must be described stochastically as well in a random system. This brings their distribution into focus. For example, it can be shown (Sibani and Hertz 1984) that if $\langle X_i\rangle<\infty$, then $P(q)=\delta(q)$, a so-called trivial Parisi function, while (Parisi 1983) in spin glasses $P(q)$ should be a smooth(er) function of its argument. We should therefore expect $\langle X_i\rangle=\infty$; however, this does not imply 'infinite clusters'. One can have a completely smooth Parisi function with clusters, all of which are finite, in the sense that the distribution of their sizes is normalised to one (Sibani and Hertz 1984). We shall assume that the cluster size distribution is stable on the basis of the following 'evidence'.

(i) A stable distribution was found in one exactly calculable example (Sibani and Hertz 1984).

(ii) Stable distributions are intimately connected with scaling laws and self-similarity arguments which are often invoked in statistical mechanics.

(iii) They possess paradoxical properties which seem well suited to describe equally paradoxical properties of spin glasses.

Since stable distributions play an important role in the present theory, and since they are not so often used in physics as they deserve, we briefly review their definition and some of their properties, in particular those related to the last remarks above.

3. Stable distributions

Following the notation of Feller (1971), to which we refer for the general theory, we write

$$U \stackrel{\mathrm{d}}{=} V, \tag{3.1}$$

to indicate that the stochastic variables U and V are identically distributed (henceforth ID). Let $X_1 \cdots X_n$ be independent ID stochastic variables with distribution U. U is stable of order α iff, for any n,

$$X_1 + \ldots + X_n = S_n \stackrel{\mathrm{d}}{=} n^{1/\alpha} X_1. \tag{3.2}$$

This means that summing any number of stably distributed variables amounts to a change of scale, a feature familiar from the Gaussian distribution, which is stable of order 2. Like the Gaussian, all stable distributions possess a domain of attraction, i.e. for ID $X'_1, \ldots, X'_n$ which satisfy some suitable condition, the scaled variable

$$Y_n = n^{1/\alpha}(X'_1 + X'_n) \tag{3.3}$$

is stably distributed in the limit $n \to \infty$. For instance, if the X'_1 have variance then $\alpha = 2$, and Y_n approaches a Gaussian variable.

Here we are interested in stable distributions of positive variables like cluster sizes and waiting times. They have the following properties (Feller 1971 chap XIII): $\alpha < 1$; a Laplace transform which, apart from scale factors is e^{-s^α}; and a domain of attraction which for a given α consists of all distributions with a tail like $1 - t^{-\alpha} L(t)$, where $L(t)$ is so-called 'slow varying'. This means by definition that $L(tx)/L(t) \to 1$ for $t \to \infty$ and x fixed. A constant and a logarithm are thus slow varying.

Each stable distribution belongs to its own domain of attraction; its density goes therefore like $t^{-(\alpha+1)}$ for large t, and its expectation diverges when $\alpha < 1$. Another unfamiliar property, which is needed in the next section, is the following: for $X_1, \ldots, X_n$ ID stochastic variables with stable distribution of order α, and $M_n = \max(X_1, \ldots, X_n)$, $S_n = (X_1 + \cdots + X_n)$, it can be shown (Feller 1971, p 465 and reference therein) that $\langle S_n / M_n \rangle \to 1/(1-\alpha)$ for $n \to \infty$, which means that the largest of the X_i's with high probability dominates the sum.

Finally, we show by two examples how stable distributions describe self-similar (fractal) systems.

(i) Consider two clusters A and B of the previous section. Their union is again a cluster $\mathrm{A} \cup \mathrm{B} = \mathrm{C}$, of size $X_\mathrm{C} = X_\mathrm{A} + X_\mathrm{B}$. The X's are stable if $X_\mathrm{C} \stackrel{\mathrm{d}}{=} 2^{1/\alpha} X_\mathrm{A}$, which means that the statistical properties of the system do not depend on the level of resolution at which it is analysed, apart from scale factors. This is the probabilistic version of scale invariance.

(ii) Recently, Palmer *et al* (1984) discussed some hierarchical constrained models for glassy relaxation, in order to explain the Kohlraush law. Similar ideas lead straightforwardly to stable distributions. Consider a set of 'spin' variables $\sigma_1, \ldots, \sigma_n$, and assume that the nth spin cannot flip unless the previous one has. (This is the constraint.) It is convenient to arrange the spins in a string of unit lattice constant, and imagine that they are flipped by a random walker according to the following rules: at time zero all the spins are up and the walker is at the origin. He waits time T_1, turns the first spin down and moves to the right. At the kth step ($k = 1, 2, \ldots$) the following alternatives arise: either go to the left and turn the $(k-1)$'s spin up again, or turn the kth spin down and move to the right. Eventually, the walker arrives at the nth spin and the whole chain is overturned in time

$$S_n = T_1 + \cdots + T_n, \tag{3.4}$$

where T_1 is the time elapsed between the flipping of spins $i-1$ and i. Our spins $\sigma_1, \ldots, \sigma_n$ describe the dynamics at a certain level of resolution, at which level the origin of the waiting times T_i remains unexplained. In a self-similar system each of the T_i's arise in the same way as the S_n above, i.e. between any two spins σ_{i-1} and σ_i a new set of variables $\sigma'_1, \ldots, \sigma'_n$ is defined, and the σ_i spin is overturned when the flipping of the primed chain is completed. However, then the T_i's are distributed just like S_n, except for time scales, and

$$S_n \overset{\mathrm{d}}{=} n^{1/\alpha} T_1, \tag{3.5}$$

which again gives stable distributions.

This kind of model will be used to describe the flipping of clusters in the next section.

4. Low-temperature dynamics

It is reasonable to believe that the ground-state morphology should have deep consequences on the low-temperature dynamics. (Here and in the following the term ground state also includes all the very low-lying states.) Most of the time the spins fluctuate in a neighbourhood of a ground state, but thermally excited transitions to another ground state may occasionally take place through the successive flipping of single (or small patches of) spins; these transitions can be described, on a coarse time scale compared to some microscopic time, as a flipping of whole clusters, without of course implying any coherent motion at a microscopic level.

In this 'coarse grained' picture, the correlation obeys

$$\langle \sigma_i(t)\sigma_j(0)\rangle = \pm\langle \sigma_i(t)\sigma_i(0)\rangle \tag{4.1}$$

if i and j are in the same irreducible cluster, and

$$\langle \sigma_i(t)\sigma_j(0)\rangle = 0 \tag{4.2}$$

otherwise.

The upper sign in equation (4.1) holds if the two spins are parallel in the ground state. Simple algebra leads now to

$$\frac{1}{N}\langle S(t)S(0)\rangle = \frac{\Sigma_c\,(2P_{X_c}(t)-1)X_c}{\Sigma_c X_c}, \tag{4.3}$$

where S is the total magnetisation, and $P_{X_c}(t)$ is the probability that a cluster of size X_c has the same orientation at times zero and t.

We assume that P_{X_c} approaches $\frac{1}{2}$ for $t \to \infty$, which means a vanishing correlation and no phase transition. Slight changes of notation give a phase transition with the same type of relaxation and AC susceptibility. However, the argument we shall use to justify the form of the relaxation function must be abandoned in the latter case. This will be discussed in full in the last section of the paper.

Based on the considerations of the previous section, we shall assume that the X_c are stably distributed which, as already noted, means that the largest of them completely dominates in (4.3). This is further enhanced by the fact that the smaller clusters relax faster, i.e. their $2P_{X_c}(t)-1$ is close to zero for large times. In conclusion, at long times only the largest cluster survives and, to a good approximation

$$(1/N)\langle S(t)S(0)\rangle \approx 2P(t)-1, \tag{4.4}$$

where the subscript on the P is henceforth omitted.

Identifying (4.4) with the EA order parameter (Edwards and Anderson 1975), and invoking the fluctuation dissipation theorem, we find that

$$\begin{aligned} T\chi_{\mathrm{AC}}(\omega) &= -\int_0^\infty \mathrm{e}^{-\mathrm{i}\omega t}\frac{\mathrm{d}}{\mathrm{d}t}\langle S(t)S(0)\rangle \\ &= 2(1-\mathrm{i}\omega\tilde{P}(\mathrm{i}\omega)), \end{aligned} \tag{4.5}$$

where $\tilde{f}(s)=\int_0^\infty \mathrm{e}^{-st}f(t)\,\mathrm{d}t$ for any function of time and any complex s such that the integral exists.

In order to calculate $P(t)$, we introduce the probability density for the nth flip at time t, $A_n(t)$, and the probability that the cluster has flipped exactly n times at time t, $P_n(t)$. Since $A_1(t)$ is the probability density that the waiting time between two jumps is t and $P_0(t)$ the probability of waiting at least time t for the next jump, we have

$$P_0(t)=\int_t^\infty A_1(z)\,\mathrm{d}z, \tag{4.6}$$

whence

$$1-s\tilde{P}_0(s)=\tilde{A}_1(s). \tag{4.7}$$

By summing probabilities, we also obtain

$$P(t)=\sum_{n=0}^{\infty} P_{2n}(t) \tag{4.8}$$

$$A_n(t)=\int_0^t A_{n-1}(t')A_1(t-t')\,\mathrm{d}t', \qquad n\geq 2 \tag{4.9}$$

$$P_n(t)=\int_0^t A_n(t')P_0(t-t')\,\mathrm{d}t', \qquad n\geq 1 \tag{4.10}$$

and, taking Laplace transforms of (4.8)-(4.10), summing a geometric series and using (4.7), we arrive at

$$\tilde{P}(s)=\frac{1}{s}\frac{1}{1+\tilde{A}_1(s)}. \tag{4.11}$$

The AC susceptibility can, by (4.5), be expressed in terms of the fundamental quantity $A_1(t)$ as

$$\chi_{\mathrm{AC}}(\omega)=\frac{2}{T}\frac{\tilde{A}_1(\mathrm{i}\omega)}{1+\tilde{A}_1(\mathrm{i}\omega)}, \tag{4.12}$$

and the power spectrum of $\langle S(t)S(0)\rangle$ is

$$\mathscr{C}(\omega)=\frac{2}{\omega}\frac{A_1''(\omega)}{(1+A_1'(\omega))^2+A_1''(\omega)^2}, \tag{4.13}$$

where $\tilde{A}(\mathrm{i}\omega)=A_1'(\mathrm{i}\omega)-\mathrm{i}A_1''(\omega)$.

Denoting the inverse Laplace transform by $\mathscr{L}^{-1}$, we also define the time-dependent susceptibility as†

$$\chi(t)=\frac{2}{T}\mathscr{L}^{-1}\left(\frac{1}{s}\frac{\tilde{A}_1(s)}{1+\tilde{A}_1(s)}\right), \tag{4.14}$$

which (by the 'Abelian theorem' $\lim_{t\to\infty}f(t)=\lim_{s\to 0}s\tilde{f}(s)$) satisfies the relation

$$\lim_{t\to\infty}\chi(t)=\lim_{\omega\to 0}\chi_{\mathrm{AC}}(\omega)=\chi_{\mathrm{eq}}=\frac{1}{T}. \tag{4.15}$$

The interesting feature of $\chi(t)$ is of course not the limiting value but the manner in which it is approached, which is 'pathologically' slow, as discussed later.

In principle $A_1(t)$ should be calculated on the basis of a microscopic description of the system. This is not feasible and might also be unnecessary. We adopt the point of view that the form of A_1 does not depend upon the details of the dynamic, but rather on some generic property of the spin glass 'phase', which we identify with self-similarity, as explained below.

We have already noted in § 2 how hierarchically constrained models naturally lead to stable distributions. Here we sketch the possible connection with spin glasses in a little more detail in order to justify the assumption that $A_1(t)$ should be stable. We start with the 'many-valley' picture of the energy as a function of the phase space variables, which is sometimes used in the literature (Palmer 1982, 1983, Morgenstern and Horner 1982, Morgenstern and Binder 1979), and which in the present context has the following meaning: in order to go from one ground state to the next we have to flip a cluster, which means climbing over a large energy barrier. A closer analysis of the barrier reveals the existence of local minima, or valleys. These valleys split again into smaller valleys, separated by barriers, and so forth through many orders of magnitude. We now fix the level of resolution, and see only a given number N of local minima, each corresponding to some configuration of the spins, which can be taken as coarse grained variables, or 'pseudospins', as explained below. The temporal sequence in which the minima are visited defines a path from one ground state to the other. If only one path (the 'easiest' one) dominates, we obtain the previously discussed constrained dynamics by translating 'pseudospin 'i' is down' into 'the ith minimum has been reached and we can proceed further'. In general, the considerations of § 2 apply to each path separately, and the flipping time associated with path A is stably distributed. At the end one should average over all the paths, which requires an adequate parametrisation of the paths and the definition of a measure. At present it is not clear how this should be done, and we just note that the 'randomisation' of the path index A and the averaging over A may be the mechanism which produces the very small order α of the stable distribution of flipping times which is needed to explain the experimental data.

† Note

$$\chi_{\mathrm{AC}}(\omega)=\int_0^\infty \mathrm{e}^{-\mathrm{i}\omega t}\frac{\mathrm{d}\chi(t)}{\mathrm{d}t}\,\mathrm{d}t.$$

5. Numerical results and comparison with the experimental results

The Laplace transform of stable distributions of positive variables is analytically known (cf § 2); in our case we have

$$\tilde{A}_1(s) = e^{-s^\alpha} \tag{5.1}$$

apart from scale factors. The temperature dependence of the parameter $\alpha(T)$ (the order of the distribution) is sometimes omitted in order to simplify the notation.

Taking

$$s = i\omega c(T), \tag{5.2}$$

where ω is the frequency and $c(T)$ is a temperature-dependent scale factor, we get by (4.12)

$$\chi_{AC}(\omega, T) = \frac{2}{T}\frac{\exp(-\omega^\alpha c(T)^\alpha \exp(i\pi\alpha/2))}{1+\exp(-\omega^\alpha c(T)^\alpha \exp(i\pi\alpha/2))}. \tag{5.3}$$

The above equation has many possible behaviours according to the magnitude of α. However, only $\alpha \ll 1$, which we assume, is consistent with the experimental findings. For reasons which will be clear shortly, we now write

$$f(T) = c(T)^{\alpha(T)} \tag{5.4}$$

and expand all the *other* functions of α in a Taylor series, keeping only up to the first-order term. Then $\omega^\alpha \approx 1 + \alpha \ln \omega$ and

$$\chi_{AC}(\omega, T) = \chi' - i\chi'' = \frac{2}{T}\frac{\exp(-f(T))}{1+\exp(-f(T))}\left(1 - \frac{f(T)\alpha(T)}{1+\exp(-f(T))}(\ln\omega + \tfrac{1}{2}i\pi)\right) \tag{5.5}$$

whence the relation

$$\chi'' = -\frac{\pi}{2}\frac{d\chi'}{d\ln\omega} \tag{5.6}$$

follows. The relation has been previously derived by phenomenological arguments and experimentally verified in a variety of systems (van Duyneveldt and Mulder 1982, Lundgren *et al* 1981, Pappa *et al* 1984). Another check on the magnitude of α comes from the fact that, according to (5.5),

$$\frac{\chi''}{\chi'} = \frac{f(T)\alpha(T)}{1+\exp(-f(T))} + O(\alpha^2).$$

This quantity is found to be extremely small ($<10^{-2}$) (Mulder *et al* 1981, 1982, Pappa *et al* 1984) at temperatures higher than T_g and rather small (2×10^{-2}-10^{-1}) around T_g. In this region, however, the dependence on $f(T)$ is significant, as will be seen later. Finally, it follows from (4.13) that the auto-correlation function is, in the same approximation as in (5.5),

$$\mathscr{C}(\omega) = \pi\alpha(T)\exp(-f(T))(1/\omega) \tag{5.7}$$

the ubiquitous $1/f$ noise. In the time domain this means a logarithmic decay of the autocorrelation function. Experimentally (Mezei 1983) the *long* time behaviour of $\langle S(t)S(0)\rangle$ (or rather its spatial Fourier transform, which does not affect the argument) seems to be of the form $k-k'\ln t$ over many decades. As an approximation, we take

$$\langle S(t)S(0)\rangle = k - 2\alpha(T)\exp(-f(T))\ln t \tag{5.8}$$

for all times such that the expression is positive, and zero otherwise. The cosine transform of (5.8) is

$$\int_0^\infty \langle S(t)S(0)\rangle \cos(\omega t)\,\mathrm{d}t = \frac{2\alpha(T)\exp(-f(T))}{\omega}\,\mathrm{Si}(\mathrm{e}^{k/\alpha}) \tag{5.9}$$

where Si is the sine integral. When α is very small $\exp(k/\alpha)$ is very large and $\mathrm{Si}(\mathrm{e}^{k/\alpha}) \approx \frac{1}{2}\pi$, yielding (5.7). Note that the slope of (5.8) in a semilog plot is $2\alpha(T)\exp(-f(T))$, which is small and increases with the temperature, in agreement with the experimental curves (Mezei 1983).

We conclude that much experimental evidence supports our assumption on the smallness of α. In order to discuss the temperature dependence of the susceptibility, the form of $\alpha(T)$ and $c(T)$ must be specified. We assume that $c(T)\sim\exp(T_0/T)$, where, according to (5.4)

$$f(T) = k\exp\left(\frac{\alpha(T)}{T}T_0\right), \tag{5.10}$$

where k is a constant.

The form of the scale factor can be justified by noting that in order to flip a cluster, a potential barrier must be crossed. $c(T)$ should therefore contain an Arrhenius factor, and a T dependent prefactor as well. This must, however, be raised to a very small power α, and behaves almost like a constant over the range of temperatures of interest. The exponential term, on the contrary, survives, and since T_0 need not be small, the shape of $\alpha(T)$ is amplified and leads to large effects. This is the reason why $c(T)^\alpha$ was not expanded in the first place.

The form of $\alpha(T)$ was found empirically by trying to fit the susceptibility of AgMn (Mulder and van Duyneveldt 1982). In (5.10) the parameters are taken as $k=0.95$, $T_0=200$, $\alpha(T)=15/100(1/T^2)$ for $T\geqslant 4.295$, and for $T<4.295$ $\alpha(T)=5/1000\,T\,(1.5146-0.2645\,T)$, a differentiable function of T with a maximum at $T\sim 3$ K. It must be emphasised that other choices of α almost surely would provide even better fits, although the overall shape should be roughly correct (but *not* for $T\leqslant 1$, where it leads to a local minimum of $\chi'(T)$). The correct $\alpha(T)$ decreases less steeply in this region. In the lack of theoretical underpinnings, we have chosen not to pursue the 'numerological' aspect any further.

Having specified all the parameters, the frequency and temperature dependence of $\chi_{\mathrm{AC}}(\omega, T)$ can be calculated through (5.3), while the relaxation of $\chi(t)$ in the time domain is described by (4.14). Since the Laplace inversion cannot be performed analytically and in closed form, we exploit the smallness of α and write $A_1(s)=\exp(-f(T)s^\alpha)\approx\exp(-f(T))(1+\alpha f(T)\ln(s))$. Expanding the denominator to first order in α and inverting the transform, we finally get (with γ equal to the Euler constant)

$$\chi(t)\approx\frac{2}{T}\frac{\exp(-f(T))}{1+\exp(-f(T))}\left(1+\frac{\alpha(T)f(T)}{1+\exp(-f(T))}(\gamma+\ln(t))\right) \tag{5.11}$$

an approximation which holds as long as $\alpha(T)\ln(t)$ is small. The limit $t=\infty$, $\chi_{\mathrm{eq}}=1/T$ cannot therefore be recovered in the above formula. Some experimental results confirming the logarithmic dependence on t are found in Lundgren *et al* (1982).

The results are graphically displayed in figures 1 and 2. In figure 1 χ' is drawn as a function of the temperature for four different frequencies: $\omega = 10$, 100, 1000 and 10^8. The results are similar to the measured χ' for AgMn (Mulder and van Duyneveldt 1982); the peak should be slightly sharper, however. We note several features quite generally observed in spin glass susceptibilities: a well defined maximum at $T_g(\omega)$, which moves to the left when ω decreases; almost no frequency dependence in the high-temperature region ($T > T_g$); the different curves separate close to their maxima. In the present framework this is due to the vanishing of $\alpha(T)$ (and $\chi''(T)$) at high temperatures, as apparent from (5.5). The lower curve in figure 1 is the out-of-phase component χ'' magnified 20 times (the absolute magnitude of χ'' and the separation between the different χ''s can be changed by rescaling α, and were chosen for graphical convenience). The shape is also similar to what is experimentally found: there is a *very* small frequency dependence (of second order in α), and as a function of

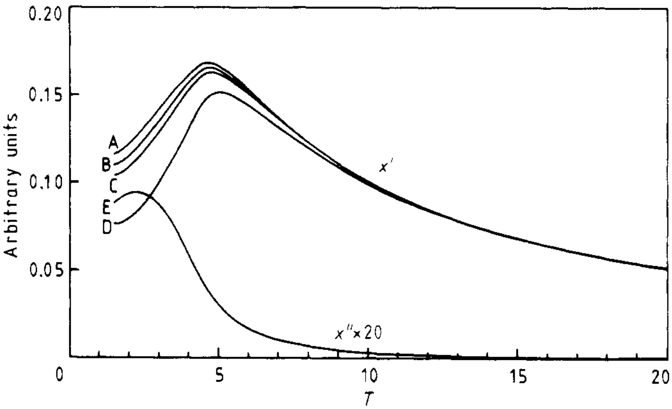

Figure 1. Curves A, B, C and D show the real part of the AC susceptibility as a function of the temperature for $\omega = 10$, 100, 1000 and $10^8\ \mathrm{s}^{-1}$, respectively. The last curve is the out-of-phase component, magnified 20 times, for $\omega = 10$. All the curves are calculated according to (5.3).

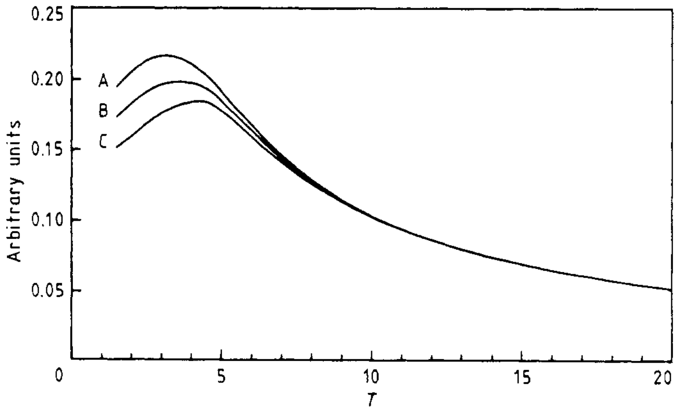

Figure 2. The time-dependent susceptibility as a function of the temperature, according to the approximate equation (5.11). Curves A, B and C correspond to $t = 10^9$, 10^6 and 10^3 s, respectively.

temperature χ'' vanishes (almost) in the high-temperature region and has a maximum somewhat below T_g. Since $\alpha(T)$ and $\chi''(T)$ are closely related, as shown by (5.5), the experimental form of χ'' was actually used to guess $\alpha(T)$. Conversely, the soundness of the choice of $\alpha(T)$ and $C(T)$ is confirmed by the fact that it yields the correct shape of *both* χ' and χ''.

In figure 2 $\chi(t, T)$, as given by (5.11), is shown as a function of T for $t = 10^3$, 10^6 and 10^9 s, at which time the validity of (5.11) begins to be questionable, since $\alpha(T) \ln t$ is not very small. The approach to equilibrium seems extremely slow: we emphasise that $\chi_{eq} = \chi_{AC}(\omega = 0) = 1/T$; nevertheless, for $t = 10^9$ s there is still a very clear maximum. The same picture holds in the frequency domain down to at least $\omega = 10^{-24}$, and we conjecture that equilibrium cannot be observed within the lifetime of the experimentalist. We also note that the maximum of $\chi(t, T)$ is more flat than in the frequency domain. Perhaps the observed 'flatness' of χ_{eq} (Lundgren 1984) can be explained within the present theory as an effect of incomplete relaxation.

6. Summary and conclusions

In this paper we propose a new theory for the dynamics of spin glasses based on the following ideas.

(i) The morphology of the ground state is responsible for the typical features of the spin glass susceptibility. The morphology is assumed to be similar in three-dimensional spin glasses and in the one- and two-dimensional models which can be analysed exactly.

(ii) To a good approximation the relaxation is described at long times by the flipping of 'clusters', i.e. groups of spins with the same relative orientation in all ground states. This is formalised by introducing the probability density for flipping at time t, $A_1(t)$, which is the crucial quantity in the dynamics.

(iii) Stable distributions are the appropriate mathematical tool to describe random self-similar systems; we assume that both cluster sizes and flipping times have stable distributions, adopting the view that spin glasses are, in several ways, 'fractal' objects.

From these assumptions the analytical formula (5.3) for χ_{AC} follows. The time dependence of the susceptibility can be found by an approximative Laplace inversion and is given by (5.11). In the formalism two unknown functions of the temperature appear: $\alpha(T)$, the order of the stable density $A_1(t)$, and a scale factor $c(T)$. There is convincing experimental evidence that $\alpha(T)$ should be very small through the whole range of T, since this leads to the correct *frequency* dependence of χ, and to the right form of the power spectrum $\mathscr{C}(\omega)$.

In order to discuss temperature dependences, $\alpha(T)$ and $c(T)$ must be specified. Close agreement with the experimental results is found by assuming that $\alpha(T)$ has a maximum somewhat below T_g, and that $c(T)$ has the Arrhenius form.

The approach to equilibrium can be studied by letting $\omega \to 0$ in (5.3), or in the time domain by taking t large in (5.11). In the last case we cannot go all the way to infinity because (5.11) is only valid as long as $\alpha \ln t \ll 1$. This includes very large times, however. In both the frequency and time domain, we find evidence that equilibrium is unattainable in experiments. Since the maximum of χ gets gradually less sharp with decreasing ω (increasing t), we conjecture that all the experimental data, including a 'flat' piece of $\chi_{eq}(T)$, can be explained within the present theory in terms of incomplete equilibrium.

This theory can be modified to include a phase transition by multiplying $A_1(t)$ by a constant $\gamma < 1$. This yields a new, defective, density of flipping times, meaning that a cluster may never turn. However, the self-similarity argument used to justify the stability of A_1 fails, since a defective density folded n times with itself approaches zero when $n \to \infty$ (see (3.4) and (3.5)). Besides, this assumption might be unnecessary, although the opinion that three-dimensional spin glasses have a phase transition seems to be favoured at the moment. Ogielski and Morgenstern (1984) recently performed a Monte Carlo simulation on a $\pm J$ model in three dimensions, finding evidence for a phase transition. If our assumption that cluster sizes are stably distributed is correct, the largest cluster has the same order of magnitude as the whole system (see § 2) and '*macroscopically*' large patches of *ordered* spins appear in a *finite* sample. In the thermodynamic limit, however, all the clusters stay finite, and so do the potential barriers which must be closed to flip them, in contrast to what, for instance, happens in a ferromagnet. The difference is rather subtle, and cannot be seen in Monte Carlo simulations of samples which, although rather large, are several orders of magnitude smaller than real spin glasses. It is quite possible that the Monte Carlo calculations of Ogielski and Morgenstern could be re-interpreted in the light of this approach. Incidentally, they do observe 'occasional coherent, rigid reversals of the entire lattice in the course of very long simulations' (Ogielski and Morgenstern 1984).

As mentioned in the introduction, our approach rests on some assumptions on the ground-state morphology which so far have only been proved for simple one- and two-dimensional models, and which should be checked for more realistic cases in order to substantiate the theory. Furthermore, a theoretical justification of the form of $\alpha(T)$ and $c(T)$ is highly desirable. Presumably, dimensionality plays an important role here.

Let us finally emphasise that this theory is the very simplest one can conceive in terms of cluster flipping; it is essentially a $T=0$ theory, in that the only effect of temperature is to induce 'tunnelling' between the different ground states. Nevertheless, it seems to capture some essential elements of the spin glass behaviour; also it is a satisfactory and unique feature of this approach that the $T=0$ equilibrium properties (non-trivial $P(q)$, clusters of the same order of magnitude as the whole system) and the slow decays are intimately connected. In the other approaches suggested, the slow relaxation is just an unexplained inconvenience that keeps one from observing the equilibrium state.

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